



V2V EDTECH LLP
Online Coaching at an Affordable Price.

OUR SERVICES:

- Diploma in All Branches, All Subjects
- Degree in All Branches, All Subjects
- BSCIT / CS
- Professional Courses

+91 93260 50669 V2V EdTech LLP
v2vedtech.com v2vedtech

YouTube Lecture Links & Notes :

Session 1 (For K & I-Scheme Students) : <https://www.youtube.com/Session-1>

Join Whtassp Group : <https://chat.whatsapp.com/DLRybi7oDaeIAcy4QR68Pw>



**SCORE 60+
IN
MATHS - 2**

I - Scheme

MSBTE FY DIPLOMA

LIVE

SUBSCRIBE

PROF. NILESH BHOSALE
10+ YEARS OF EXPERIENCE

Integration

1) $\int 0 \, dx = \text{constant}$	11) $\int \sin x \, dx = -\cos x + C$
2) $\int 1 \, dx = x + C$	12) $\int \cos x \, dx = \sin x + C$
3) $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$	13) $\int \tan x \, dx = \log \sec x + C$
4) $\int e^x \, dx = \frac{e^x}{1} + C$	14) $\int \cot x \, dx = \log \tan x + C$
5) $\int e^x \, dx = e^x + C$	15) $\int \sec x \, dx = \log \sec x + \tan x + C$
6) $\int \frac{1}{x} \, dx = \log x + C$	16) $\int \csc x \, dx = \log \csc x - \cot x + C$
7) $\int \frac{1}{\sin x} \, dx = \log \tan x + \sec x + C$	17) $\int \sec^2 x \, dx = \tan x + C$
8) $\int \log x \, dx = x \log x - x + C$	18) $\int \csc^2 x \, dx = -\cot x + C$
9) $\int (a^x \cdot b^x) \cdot c^x \, dx = \frac{a^x \cdot b^x \cdot c^x}{\log a + \log b + \log c} + C$	19) $\int \sec x \cdot \tan x \, dx = \sec x + C$
10) $\int \frac{1}{x^2} \, dx = -\frac{1}{x} + C$	20) $\int \csc x \cdot \cot x \, dx = -\csc x + C$

* $\int \frac{f'(x)}{f(x)} \, dx = \log |f(x)| + C$

$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C$	$\int \frac{-1}{\sqrt{1-x^2}} \, dx = \cos^{-1} x + C$
$\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$	$\int \frac{-1}{1+x^2} \, dx = \cot^{-1} x + C$
$\int \frac{x}{x\sqrt{x^2-1}} \, dx = \sec^{-1} x + C$	$\int \frac{-x}{x\sqrt{x^2-1}} \, dx = \csc^{-1} x + C$

1 sum / 4 Marks

* $\int \frac{1}{x^2+a^2} \, dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$

* $\int \frac{1}{x^2-a^2} \, dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$

* $\int \frac{1}{a^2-x^2} \, dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$

$\int u \cdot v \, dx = u \int v \, dx - \int \left[\frac{d}{dx} u \cdot \int v \, dx \right] dx$

L = Logarithmic = $\log x$
 I = Inverse = $\sin^{-1} x / \tan^{-1} x / \cos^{-1} x \dots$
 A = Algebraic = $1, x, x^2 \dots$
 T = Trigonometric = $\sin x / \cos x / \tan x \dots$
 E = Exponential = $e^x / e^{2x} / e^{3x} \dots$

$\text{Area} = \int_a^b y \, dx$

$\text{Area} = \int_a^b (y_1 - y_2) \, dx$

* $\int_a^b f(x) \, dx = \int_a^b f(a+b-x) \, dx$

Derivative

$\frac{d}{dx} \text{constant} = 0$	$\frac{d}{dx} \sin x = \cos x$
$\frac{d}{dx} x^n = n \cdot x^{n-1}$	$\frac{d}{dx} \cos x = -\sin x$
$\frac{d}{dx} k \cdot x^n = k \cdot \frac{d}{dx} x^n$	$\frac{d}{dx} \tan x = \sec^2 x$
$\frac{d}{dx} a^x = a^x \cdot \log_e a$	$\frac{d}{dx} \cot x = -\csc^2 x$
$\frac{d}{dx} e^x = e^x$	$\frac{d}{dx} \sec x = \sec x \cdot \tan x$
$\frac{d}{dx} \log x = \frac{1}{x}$	$\frac{d}{dx} \csc x = -\csc x \cdot \cot x$

① $\frac{d}{dx} (u \pm v \mp w) = \frac{d}{dx} u \pm \frac{d}{dx} v \mp \frac{d}{dx} w$

② Product Rule
 $\frac{d}{dx} (u \cdot v) = u \cdot \frac{d}{dx} v + v \cdot \frac{d}{dx} u$

③ Division Rule
 $\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \cdot \frac{d}{dx} u - u \cdot \frac{d}{dx} v}{v^2}$

④ Chain Rule
 $\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx}$

$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$
$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$	$\frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2}$
$\frac{d}{dx} \sec^{-1} x = \frac{1}{ x \sqrt{x^2-1}}$	$\frac{d}{dx} \csc^{-1} x = \frac{-1}{ x \sqrt{x^2-1}}$

$$\frac{a}{0} = \infty$$

$$\frac{0}{a} = 0$$

$$a^0 = 1$$

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

$$a^x \times b^x = (ab)^x$$

$$\frac{1}{x^1} = x^{-1}$$

$$\frac{1}{x^3} = x^{-3}$$

$$\frac{1}{x^{7/2}} = x^{-7/2}$$

$$\frac{1}{x^{-9/4}} = x^{9/4}$$

$$\sqrt{x} = x^{1/2}$$

$$\sqrt[3]{x} = x^{1/3}$$

$$4\sqrt{x} = 4 \cdot x^{1/2}$$

$$Q. I = \int \frac{1}{\underline{3x+4}} dx$$

$$\Rightarrow \int \frac{1}{\underline{ax+b}} dx = \log |ax+b| \cdot \frac{1}{a} + C$$

$$\therefore I = \log |3x+4| \cdot \frac{1}{3} + C //$$

$$Q. I = \int \frac{1}{4-5x} dx$$

$$\int \frac{1}{\underline{ax+b}} dx = \log |ax+b| \cdot \frac{1}{a} + C$$

$$\therefore I = \log |4-5x| \cdot \frac{1}{(-5)} + C //$$

$$\begin{array}{l|l}
 \textcircled{*} \sin 2A = 2 \sin A \cdot \cos A & \textcircled{*} \cos 2A = \cos^2 A - \sin^2 A \\
 \textcircled{*} = \frac{2 \tan A}{1 + \tan^2 A} & \textcircled{*} = 2 \cos^2 A - 1 \\
 & \textcircled{*} = 1 - 2 \sin^2 A \\
 & = \frac{1 - \tan^2 A}{1 + \tan^2 A}
 \end{array}
 \quad \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin 3A = 3 \sin A - 4 \sin^3 A$$

$$\cos 3A = 4 \cos^3 A - 3 \cos A$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

$$\begin{aligned}
 \text{Q. } I &= \int (a^n + a^x + x^n + e^x) dx \\
 \Rightarrow &= \int a^n dx + \int a^x dx + \int x^n dx + \int e^x dx \\
 &= a^n \int 1 \cdot dx + \frac{a^x}{\log_e a} + \frac{x^{n+1}}{n+1} + e^x + C \\
 I &= a^n(x) + \frac{a^x}{\log_e a} + \frac{x^{n+1}}{n+1} + e^x + C //
 \end{aligned}$$

$$\begin{aligned}
 \text{Q. } I &= \int (\underline{a^{4x}} + e^{5x} + x^6 + 7) dx \\
 &= \int a^{4x} dx + \int e^{5x} dx + \int x^6 dx + \int 7 \cdot dx \\
 \left. \begin{aligned} \int a^x dx &= \frac{a^x}{\log_e a} + C \\ \int a^{4x} dx &= \frac{a^{4x}}{\log_e a} \cdot \frac{1}{4} + C \end{aligned} \right| & \left. \begin{aligned} \int e^x dx &= e^x + C \\ \int e^{5x} dx &= e^{5x} \cdot \frac{1}{5} + C \end{aligned} \right| & \left. \begin{aligned} \int x^n dx &= \frac{x^{n+1}}{n+1} + C \\ \int x^6 dx &= \frac{x^{6+1}}{6+1} + C \\ &= \frac{x^7}{7} + C // \end{aligned} \right| & \begin{aligned} \int 7 \cdot dx &= 7 \int 1 \cdot dx \\ &= 7x + C // \end{aligned} \\
 \therefore I &= \frac{a^{4x}}{\log_e a} \cdot \frac{1}{4} + e^{5x} \cdot \frac{1}{5} + \frac{x^7}{7} + 7x + C //
 \end{aligned}$$

$$\begin{aligned}
 \text{Q. } I &= \int x(x-1)^2 dx \\
 \Rightarrow (a-b)^2 &= a^2 - 2ab + b^2 \\
 (x-1)^2 &= x^2 - 2x(1) + 1^2 \\
 (x-1)^2 &= x^2 - 2x + 1 \\
 \therefore I &= \int x(x^2 - 2x + 1) \cdot dx \\
 &= \int (x^3 - 2x^2 + x) dx \\
 &= \int x^3 dx - \int 2x^2 dx + \int x dx \\
 \int x^n dx &= \frac{x^{n+1}}{n+1} + C \\
 \therefore I &= \frac{x^{3+1}}{3+1} - 2 \int x^2 dx + \frac{x^{1+1}}{1+1} + C \\
 I &= \frac{x^4}{4} - 2 \frac{x^3}{3} + \frac{x^2}{2} + C //
 \end{aligned}$$

$$\begin{aligned}
 \text{Q. } I &= \int \sin^2 x dx \\
 &= \int \frac{1}{2} (1 - \cos 2x) dx \\
 &= \frac{1}{2} \int (1 - \cos 2x) dx \\
 &= \frac{1}{2} \left[\int 1 \cdot dx - \int \cos 2x dx \right] \\
 &= \frac{1}{2} \left[x - \sin 2x \cdot \frac{1}{2} \right] + C //
 \end{aligned}$$

$$\begin{aligned}
 \cos 2A &= \cos^2 A - \sin^2 A \\
 \cos 2A &= 1 - 2\sin^2 A \\
 2\sin^2 A &= 1 - \cos 2A \\
 \sin^2 A &= \frac{1}{2} (1 - \cos 2A) \\
 \int \cos x dx &= \sin x + C \\
 \int \cos 2x dx &= (\sin 2x) \frac{1}{2} + C
 \end{aligned}$$

$$Q. I = \int \frac{x}{(x+2)(x+3)} dx$$

$$\Rightarrow \frac{x}{(x+2)(x+3)} = \frac{A}{(x+2)} + \frac{B}{(x+3)}$$

$$x = A(x+3) + B(x+2) \quad \text{--- (1)}$$

$$\left. \begin{array}{l} x+3=0 \\ x=-3 \end{array} \right\} \text{eqn (1)} \Rightarrow (-3) = A[(-3)+3] + B[(-3)+2]$$

$$(-3) = A(0) + B(-1)$$

$$(-3) = B(-1)$$

$$\therefore \frac{(-3)}{(-1)} = \boxed{B = 3}$$

$$\left. \begin{array}{l} x+2=0 \\ x=-2 \end{array} \right\} \text{eqn (1)} \Rightarrow (-2) = A[(-2)+3] + B[(-2)+2]$$

$$(-2) = A[1] + B(0)$$

$$\boxed{(-2) = A}$$

$$\frac{x}{(x+2)(x+3)} = \frac{A}{(x+2)} + \frac{B}{(x+3)}$$

$$\frac{x}{(x+2)(x+3)} = \frac{(-2)}{(x+2)} + \frac{3}{(x+3)}$$

$$I = \int \frac{x}{(x+2)(x+3)} dx = \int \left[\frac{(-2)}{(x+2)} + \frac{3}{(x+3)} \right] dx$$

$$= \int \frac{(-2)}{(x+2)} dx + \int \frac{3}{(x+3)} dx$$

$$= (-2) \int \frac{1}{(x+2)} dx + 3 \int \frac{1}{(x+3)} dx$$

$$\int \frac{1}{ax+b} dx = \log|ax+b| \cdot \frac{1}{a} + C$$

$$I = (-2) \cdot \log|x+2| \cdot \frac{1}{1} + 3 \cdot \log|x+3| \cdot \frac{1}{1} + C$$

$$I = (-2) \cdot \log|x+2| + 3 \cdot \log|x+3| + C //$$

$$Q. I = \int \frac{1}{x+5} dx$$

$$\int \frac{1}{ax+b} dx = \log|ax+b| \cdot \frac{1}{a} + C$$

$$I = \log|x+5| \cdot \frac{1}{1} + C$$

$$I = \log|x+5| + C$$

$$Q. I = \int \frac{x}{x+5} dx$$

$$I = \int \frac{(x+5)-5}{(x+5)} dx$$

$$= \int \left[\frac{\cancel{(x+5)}}{\cancel{(x+5)}} - \frac{5}{(x+5)} \right] dx$$

$$= \int \left[1 - \frac{5}{(x+5)} \right] dx$$

$$= \int 1 \cdot dx - \int \frac{5}{(x+5)} dx$$

$$= x - 5 \int \frac{1}{x+5} dx + C$$

$$= x - 5 \cdot \log|x+5| \cdot \frac{1}{1} + C$$

$$I = x - 5 \cdot \log|x+5| + C //$$

$$I = \int \frac{x}{x^2+5} dx$$

$$f(x) = x^2+5$$

diff. w.r.t x

$$f'(x) = 2x+0$$

$$= 2x$$

$$I = \frac{1}{2} \int \frac{2x}{x^2+5} dx$$

$$\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$$

$$\therefore I = \frac{1}{2} \cdot \log|x^2+5| + C //$$

Third term sum

Q. $I = \int \frac{dx}{1x^2 + 4x + 25}$

First term = FT = 1

Middle term = MT = 4

$$\text{Third Term} = \frac{(MT)^2}{4 \times FT}$$

$$= \frac{4^2}{4 \times 1}$$

$$\text{Third term} = 4$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$\therefore I = \int \frac{dx}{x^2 + 4x + 4 - 4 + 25}$$

$$a^2 = x^2$$

$$a = x$$

$$b^2 = 4$$

$$b = 2$$

$$\therefore I = \int \frac{dx}{(x+2)^2 + 21}$$

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \cdot \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$x = (x+2) \quad a^2 = 21$$

$$a = \sqrt{21}$$

$$\therefore I = \frac{1}{\sqrt{21}} \cdot \tan^{-1}\left(\frac{x+2}{\sqrt{21}}\right) + C //$$

u.v
sum

$$I = \int x \cdot \tan^{-1} x \, dx$$

$$\int u \cdot v \, dx = u \int v \, dx - \int \left[\frac{d}{dx} u \cdot \int v \, dx \right] dx$$

$$L =$$

$$I = \tan^{-1} x \Rightarrow u$$

$$A = x \Rightarrow v$$

$$T =$$

$$E =$$

$$\therefore I = \int \tan^{-1} x \cdot x \, dx = \tan^{-1} x \int x' \, dx - \int \left[\frac{d}{dx} \tan^{-1} x \cdot \int x \, dx \right] dx$$

$$= \tan^{-1} x \cdot \left[\frac{x^2}{2} \right] - \int \left[\frac{1}{1+x^2} \cdot \frac{x^2}{2} \right] \cdot dx + C$$

$$= \tan^{-1} x \cdot \left(\frac{x^2}{2} \right) - \frac{1}{2} \int \left[\frac{x^2}{1+x^2} \right] \cdot dx + C$$

$$= \frac{x^2}{2} \cdot \tan^{-1} x - \frac{1}{2} \int \left[\frac{(1+x^2) - 1}{(1+x^2)} \right] dx + C$$

$$= \frac{x^2}{2} \cdot \tan^{-1} x - \frac{1}{2} \int \left[\frac{\cancel{(1+x^2)}^1}{\cancel{(1+x^2)}^1} - \frac{1}{(1+x^2)} \right] dx + C$$

$$= \frac{x^2}{2} \cdot \tan^{-1} x - \frac{1}{2} \int \left[1 - \frac{1}{(1+x^2)} \right] dx + C$$

$$= \frac{x^2}{2} \cdot \tan^{-1} x - \frac{1}{2} \left[\int 1 \, dx - \int \frac{1}{1+x^2} \, dx \right] + C$$

$$I = \frac{x^2}{2} \cdot \tan^{-1} x - \frac{1}{2} [x - \tan^{-1} x] + C //$$

$$I = \int \log x \, dx$$

$$\Rightarrow I = \int 1 \cdot \log x \, dx$$

$$I = \int x \cdot e^x \, dx$$

$$\int u \cdot v \, dx = u \int v \, dx - \int \left[\frac{d}{dx} u \cdot \int v \, dx \right] dx$$

$$\begin{array}{l} L \\ I \\ A \\ T \\ E \end{array} = \log x = u$$

$$A = 1 \Rightarrow v$$

$$\int \log x \cdot 1 \, dx = \log x \int 1 \cdot dx - \int \left[\frac{d}{dx} \log x \cdot \int 1 \, dx \right] dx$$

$$= \log x [x] - \int \left[\frac{1}{x} \cdot x \right] dx + C$$

$$= x \cdot \log x - \int 1 \cdot dx + C$$

$$I = x \cdot \log x - x + C //$$

Q.11 Definite
Integration

$$I = \int_{a=2}^{b=5} \frac{\sqrt{x}}{\sqrt{7-x} + \sqrt{x}} dx \quad \text{--- ①}$$

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$(x) \Rightarrow (a+b-x)$$

$$(x) \Rightarrow (2+5-x) = (7-x)$$

$$\therefore \text{eqn ①} \Rightarrow I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-(7-x)} + \sqrt{7-x}} dx$$

$$= \int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-7+x} + \sqrt{7-x}} dx$$

$$I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{x} + \sqrt{7-x}} dx \quad \text{--- ②}$$

$$I = \int_{a=2}^{b=5} \frac{\sqrt{x}}{\sqrt{7-x} + \sqrt{x}} dx \quad \text{--- ①}$$

$$\text{eqn ②} + \text{①}$$

$$I + I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{x} + \sqrt{7-x}} dx + \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x} + \sqrt{x}} dx$$

$$2I = \int_2^5 \frac{\cancel{(\sqrt{7-x} + \sqrt{x})}}{\cancel{(\sqrt{x} + \sqrt{7-x})}} dx$$

$$2I = \int_2^5 1 dx$$

$$2I = [x]_2^5$$

$$2I = [5 - 2]$$

$$2I = 3$$

$$I = \frac{3}{2} //$$

Substitution
Method

$$I = \int \frac{1}{2+3\cos 2x} \cdot dx$$

$\Rightarrow$

$$\begin{aligned} \tan x &= t \\ \cos 2x &= \frac{1-t^2}{1+t^2} \\ dx &= \frac{1}{1+t^2} dt \\ \sin 2x &= \frac{2t}{1+t^2} \end{aligned}$$

$$I = \int \frac{1}{2+3\left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{1}{(1+t^2)} dt$$

$$\therefore I = \int \frac{1}{\frac{2(1+t^2)+3(1-t^2)}{1+t^2}} \cdot \frac{dt}{\cancel{(1+t^2)}}$$

$$\therefore I = \int \frac{1}{2(1+t^2)+3(1-t^2)} \cdot dt$$

$$= \int \frac{1}{2+2t^2+3-3t^2} \cdot dt$$

$$I = \int \frac{1}{5-t^2} \cdot dt$$

$$\int \frac{1}{a^2-x^2} = \frac{1}{2a} \cdot \log \left| \frac{a+x}{a-x} \right| + C$$

$$\begin{aligned} a^2 &= 5 & x^2 &= t^2 \\ a &= \sqrt{5} & x &= t \end{aligned}$$

$$\therefore I = \frac{1}{2\sqrt{5}} \cdot \log \left| \frac{\sqrt{5}+t}{\sqrt{5}-t} \right| + C$$

$$= \frac{1}{2\sqrt{5}} \cdot \log \left| \frac{\sqrt{5}+\tan x}{\sqrt{5}-\tan x} \right| + C$$

$$I = \int \frac{1}{2+3\cos x} dx$$

$\Rightarrow$

$$\begin{aligned} \tan\left(\frac{x}{2}\right) &= t \\ \cos x &= \frac{1-t^2}{1+t^2} \\ dx &= \frac{2dt}{1+t^2} \\ \sin x &= \frac{2t}{1+t^2} \end{aligned}$$

$$I = \int \frac{1}{2+3\cos x} \cdot dx = \int \frac{1}{2+3\left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{2dt}{(1+t^2)}$$

$$\therefore I = \int \frac{1}{\frac{2(1+t^2)+3(1-t^2)}{1+t^2}} \cdot \frac{2dt}{\cancel{(1+t^2)}}$$

$$= 2 \int \frac{1}{2(1+t^2)+3(1-t^2)} \cdot dt$$

$$= 2 \int \frac{1}{\underline{2+2t^2+3-3t^2}} dt$$

$$= 2 \int \frac{1}{5-t^2} dt$$

$$\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \cdot \log \left| \frac{a+x}{a-x} \right| + C$$

$$\begin{aligned} a^2 &= 5 & x^2 &= t^2 \\ \therefore a &= \sqrt{5} & x &= t \end{aligned}$$

$$\therefore I = \cancel{2} \left[\frac{1}{2\sqrt{5}} \cdot \log \left| \frac{\sqrt{5}+t}{\sqrt{5}-t} \right| \right] + C$$

$$I = \frac{1}{\sqrt{5}} \cdot \log \left| \frac{\sqrt{5}+\tan(x/2)}{\sqrt{5}-\tan(x/2)} \right| + C //$$

Partial Fraction

$$I = \int \frac{1}{x(2-\log x)(2\log x-1)} dx$$

$\log x = t$
 $\frac{d}{dx} \log x = \frac{1}{x}$
 $\frac{1}{x} dx = dt$

$$I = \int \frac{1}{(2-t)(2t-1)} dt$$

$$\frac{1}{(2-t)(2t-1)} = \frac{A}{(2-t)} + \frac{B}{(2t-1)}$$

$$1 = A(2t-1) + B(2-t)$$

$2t-1=0 \Rightarrow t=\frac{1}{2}$
 $1 = A(2 \cdot \frac{1}{2} - 1) + B(2 - \frac{1}{2})$
 $1 = A(0) + B(\frac{3}{2})$
 $1 = B(\frac{3}{2}) \Rightarrow B = \frac{2}{3}$

$2-t=0 \Rightarrow t=2$
 $1 = A(2 \cdot 2 - 1) + B(2 - 2)$
 $1 = A(3) + B(0)$
 $1 = A(3)$
 $A = \frac{1}{3}$

$$\frac{1}{(2-t)(2t-1)} = \frac{1/3}{(2-t)} + \frac{2/3}{(2t-1)}$$

$$\therefore I = \int \frac{1}{(2-t)(2t-1)} dt$$

$$= \int \left[\frac{1/3}{(2-t)} + \frac{2/3}{(2t-1)} \right] dt$$

$$= \int \frac{1/3}{(2-t)} dt + \int \frac{2/3}{(2t-1)} dt$$

$$= \frac{1}{3} \int \frac{1}{2-t} dt + \frac{2}{3} \int \frac{1}{2t-1} dt$$

$$\int \frac{1}{ax+b} dx = \log|ax+b| \cdot \frac{1}{a} + C$$

$$\therefore I = \frac{1}{3} \cdot \log|2-t| \cdot \frac{1}{(-1)} + \frac{2}{3} \cdot \log|2t-1| \cdot \frac{1}{2} + C$$

$$I = -\frac{1}{3} \cdot \log|2-t| + \frac{1}{3} \log|2t-1| + C //$$

$$\therefore I = -\frac{1}{3} \cdot \log|2-\log x| + \frac{1}{3} \log|2\log x-1| + C //$$

Q. $I = \int \frac{e^x}{(e^x-1)(e^x+1)} dx$

$$\therefore I = \int \frac{1}{(t-1)(t+1)} dt$$

$$\frac{1}{(t-1)(t+1)} = \frac{A}{(t-1)} + \frac{B}{(t+1)}$$

$$1 = A(t+1) + B(t-1) \text{ --- (1)}$$

$e^x = t$
 differentiating w.r.t 'x'
 $\frac{d}{dx} e^x = \frac{d}{dx} t$
 $e^x = \frac{dt}{dx}$
 $e^x dx = dt$

$t+1=0 \Rightarrow t=-1$
 $1 = A[(-1)+1] + B[(-1)-1]$
 $1 = A(0) + B(-2)$
 $1 = B(-2)$
 $\frac{1}{(-2)} = B \Rightarrow B = -1/2$

$t-1=0 \Rightarrow t=1$
 $1 = A(1+1) + B(1-1)$
 $1 = A(2) + B(0)$
 $1 = A(2)$
 $\frac{1}{2} = A$

$$= \frac{A}{(t-1)} + \frac{B}{(t+1)}$$

$$\frac{1}{(t-1)(t+1)} = \frac{1/2}{(t-1)} + \frac{(-1/2)}{(t+1)}$$

$$\therefore I = \int \frac{1}{(t-1)(t+1)} dt$$

$$= \int \left[\frac{1/2}{(t-1)} + \frac{(-1/2)}{(t+1)} \right] dt$$

$$= \int \frac{1/2}{(t-1)} dt + \int \frac{(-1/2)}{(t+1)} dt$$

$$= \frac{1}{2} \int \frac{1}{(t-1)} dt + \left(-\frac{1}{2} \right) \int \frac{1}{(t+1)} dt$$

$$\int \frac{1}{ax+b} dx = \log|ax+b| \cdot \frac{1}{a} + C$$

$$\therefore I = \frac{1}{2} \log|t-1| \cdot \frac{1}{1} + \left(-\frac{1}{2} \right) \cdot \log|t+1| \cdot \frac{1}{1} + C //$$

$$= \frac{1}{2} \log|e^x-1| - \frac{1}{2} \log|e^x+1| + C$$

$$= \frac{1}{2} \left[\log|e^x-1| - \log|e^x+1| \right] + C$$

$$I = \frac{1}{2} \log \frac{|e^x-1|}{|e^x+1|} + C //$$

Substitution
Method

$$I = \int \frac{e^x(x+1)}{\cos^2(x \cdot e^x)} dx$$

$$I = \int \frac{1}{\cos^2 t} dt$$

$$I = \int \sec^2 t \cdot dt$$

$$= \tan t + C$$

$$I = \tan(x \cdot e^x) + C$$

$$x \cdot e^x = t$$

differentiating w.r.t 'x'

$$\frac{d}{dx}(x \cdot e^x) = \frac{d}{dx} t$$

$$u \cdot \frac{d}{dx} v + v \cdot \frac{d}{dx} u = \frac{dt}{dx}$$

$$x \cdot \frac{d}{dx} e^x + e^x \cdot \frac{d}{dx} x = \frac{dt}{dx}$$

$$x \cdot e^x + e^x \cdot 1 = \frac{dt}{dx}$$

$$e^x(x+1) = \frac{dt}{dx}$$

$$e^x(x+1) \cdot dx = dt //$$

2 Marks

Q. find degree & order

$$\sqrt{\frac{d^2y}{dx^2}} - \frac{dy}{dx} - xy^2 = 0$$

⇒

Order = 2

$$\sqrt{\frac{d^2y}{dx^2}} - \frac{dy}{dx} - xy^2 = 0$$

$$\sqrt{\frac{d^2y}{dx^2}} = xy^2 + \frac{dy}{dx}$$

$$\left(\frac{d^2y}{dx^2}\right)^{1/2} = \left(xy^2 + \frac{dy}{dx}\right)$$

squaring both sides

$$\left[\left(\frac{d^2y}{dx^2}\right)^{1/2}\right]^2 = \left(xy^2 + \frac{dy}{dx}\right)^2$$

$$\left(\frac{d^2y}{dx^2}\right)^1 = \left(xy^2 + \frac{dy}{dx}\right)^2$$

degree = 1

order = 2

Q. find Order & degree

$$\sqrt[3]{\frac{dy}{dx} + y} = \sqrt[4]{\frac{d^2y}{dx^2}}$$

⇒

$$\left(\frac{dy}{dx} + y\right)^{1/3} = \left(\frac{d^2y}{dx^2}\right)^{1/4}$$

taking cube on both sides

$$\left[\left(\frac{dy}{dx} + y\right)^{1/3}\right]^3 = \left[\left(\frac{d^2y}{dx^2}\right)^{1/4}\right]^3$$

$$\left(\frac{dy}{dx} + y\right)^1 = \left(\frac{d^2y}{dx^2}\right)^{3/4}$$

taking raise 4 both side

$$\left[\left(\frac{dy}{dx} + y\right)^1\right]^4 = \left[\left(\frac{d^2y}{dx^2}\right)^{3/4}\right]^4$$

$$\left(\frac{dy}{dx} + y\right)^1 = \left(\frac{d^2y}{dx^2}\right)^3$$

Order = 2

degree = 3

4 Marks

Q. If $y = A \sin x + B \cos x$
show that $\frac{d^2 y}{dx^2} + y = 0$

$\Rightarrow y = A \sin x + B \cos x$ ——— ①

differentiating w.r.t x

$$\frac{d}{dx} y = \frac{d}{dx} (A \sin x + B \cos x)$$

$$= \frac{d}{dx} A \sin x + \frac{d}{dx} B \cos x$$

$$= A \cdot \frac{d}{dx} \sin x + B \cdot \frac{d}{dx} \cos x$$

$$= A (\cos x) + B (-\sin x)$$

$$\frac{dy}{dx} = A \cos x - B \sin x$$

differentiating w.r.t ' x '

$$\frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} (A \cos x - B \sin x)$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dx} A \cos x - \frac{d}{dx} B \sin x$$

$$= A \frac{d}{dx} \cos x - B \frac{d}{dx} \sin x$$

$$= A (-\sin x) - B (\cos x)$$

$$\frac{d^2 y}{dx^2} = -A \sin x - B \cos x$$

$$\frac{d^2 y}{dx^2} = - (A \sin x + B \cos x)$$

$$\frac{d^2 y}{dx^2} = -y$$

$$\frac{d^2 y}{dx^2} + y = 0 \quad // \text{ Hence proved } //$$

4 Marks

Q. Remove Arbitrary Constant

$$y^2 = 4ax$$

$$\Rightarrow y^2 = 4ax$$

differentiating w.r.t 'x'

$$\frac{d}{dx} y^2 = \frac{d}{dx} 4ax$$

$$\frac{d}{dx} x^2 = 4a \frac{d}{dx} x$$

$$2x \cdot \frac{dy}{dx} = 4a(1)$$

$$2y \cdot \frac{dy}{dx} = 4a \quad \text{--- ①}$$

$$y^2 = 4ax$$

$$\frac{y^2}{2} = 4a$$

$$\therefore \text{eqn ①} \Rightarrow 2y \cdot \frac{dy}{dx} = \frac{y^2}{x}$$

$$2y \cdot \frac{dy}{dx} - \frac{y^2}{x} = 0 //$$

$\therefore$ Required D.E

Q. Remove Arbitrary Constant

$$\text{if } y = A \cos 3x + B \sin 3x$$

$$\Rightarrow y = A \cos 3x + B \sin 3x \quad \text{--- ①}$$

differentiating w.r.t 'x'

$$\frac{d}{dx} y = \frac{d}{dx} (A \cos 3x + B \sin 3x)$$

$$= \frac{d}{dx} A \cos 3x + \frac{d}{dx} B \sin 3x$$

$$= A \frac{d}{dx} \cos 3x + B \frac{d}{dx} \sin 3x$$

$$= A (-\sin x) \frac{d}{dx} x + B (\cos x) \cdot \frac{d}{dx} x$$

$$= A (-\sin 3x) \frac{d}{dx} 3x + B \cos 3x \cdot \frac{d}{dx} 3x$$

$$\frac{dy}{dx} = -A \sin 3x \cdot 3 \frac{d}{dx} x + B \cos 3x \cdot 3 \frac{d}{dx} x$$

$$\frac{dy}{dx} = -A \sin 3x \cdot 3(1) + B \cos 3x \cdot 3(1)$$

$$\frac{dy}{dx} = -3A \sin 3x + 3B \cos 3x$$

differentiating w.r.t 'x'

$$\frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} (-3A \sin 3x + 3B \cos 3x)$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dx} (-3A \sin 3x) + \frac{d}{dx} (3B \cos 3x)$$

$$= -3A \cdot \frac{d}{dx} \sin 3x + 3B \frac{d}{dx} \cos 3x$$

$$= -3A \cdot \cos x \cdot \frac{d}{dx} x + 3B \cdot (-\sin x) \cdot \frac{d}{dx} x$$

$$= -3A \cdot \cos 3x \cdot \frac{d}{dx} 3x + 3B (-\sin 3x) \cdot \frac{d}{dx} 3x$$

$$\frac{d^2 y}{dx^2} = -3A \cdot \cos 3x \cdot 3(1) - 3B \sin 3x \cdot 3(1)$$

$$\frac{d^2 y}{dx^2} = -9A \cos 3x - 9B \sin 3x$$

$$\frac{d^2 y}{dx^2} = -9(A \cos 3x + B \sin 3x) \quad \text{--- eqn ①}$$

$$\frac{d^2 y}{dx^2} = -9y$$

$$\frac{d^2 y}{dx^2} + 9y = 0 //$$

Required D.E //

4 Marks

Q. Solve D.E

$$1) \frac{dy}{dx} + y \cdot \cot x = \operatorname{cosec} x$$

$$\Rightarrow \text{step ①} \quad \frac{dy}{dx} + Py = Q$$

$$P = \cot x$$

$$Q = \operatorname{cosec} x$$

$$\text{step ②} \quad \text{IF} = \text{Integrated factor} \\ = e^{\int P \cdot dx}$$

$$\int P \cdot dx = \int \cot x \cdot dx$$

$$\int P \cdot dx = \log |\sin x|$$

$$\text{IF} = e^{\int P \cdot dx}$$

$$\text{IF} = e^{\log |\sin x|}$$

$$e^{\log x} = x$$

$$\therefore \text{IF} = \sin x$$

$$\text{step ③} \quad y \cdot \text{IF} = \int Q \cdot \text{IF} \cdot dx + C$$

$$y \cdot \sin x = \int \operatorname{cosec} x \cdot \sin x \cdot dx + C$$

$$= \int \frac{1}{\sin x} \cdot \sin x \cdot dx + C$$

$$= \int 1 \cdot dx + C$$

$$y \cdot \sin x = x + C$$

$$y \cdot \sin x - x - C = 0 //$$

Q. Solve D.E

$$2) x \cdot \frac{dy}{dx} - y = x^2 \text{ --- ①}$$

$$\Rightarrow \text{step ①} \quad \frac{dy}{dx} + Py = Q$$

dividing eqn ① by 'x'

$$\frac{x \cdot \frac{dy}{dx} - y}{x} = \frac{x^2}{x}$$

$$\frac{dy}{dx} - y \left(\frac{1}{x} \right) = \left(x \right)$$

$$P = x^{-1}$$

$$Q = x$$

step ② Find Integrated factor

$$\text{IF} = e^{\int P \cdot dx}$$

$$\int P \cdot dx = \int x^{-1} \cdot dx$$

$$= \int \frac{1}{x} \cdot dx$$

$$\int P \cdot dx = \log x$$

$$\therefore \text{IF} = e^{\log x}$$

$$e^{\log x} = x$$

$$\therefore \text{IF} = x$$

$$\text{step ③} \quad y \cdot \text{IF} = \int Q \cdot \text{IF} \cdot dx + C$$

$$y \cdot x = \int x \cdot x \cdot dx + C$$

$$xy = \int x^2 \cdot dx + C$$

$$xy = \frac{x^3}{3} + C$$

$$xy - \frac{x^3}{3} - C = 0 //$$

2 Marks

Q. find area bounded by
 $y = x^2$ & x-axis
from $x=0$ to $x=3$

$$\begin{aligned}\Rightarrow \text{Area} &= \int_a^b y \cdot dx \\ &= \int_{a=0}^{b=3} x^2 \cdot dx \\ &= \left[\frac{x^3}{3} \right]_0^3 \\ &= \left[\frac{3^3}{3} - \frac{0^3}{3} \right] \\ &= \left[\frac{27}{3} - 0 \right] \\ \text{Area} &= 9 \text{ sq. units,}\end{aligned}$$

Q. find area bounded by $y = \sin x$ & x-axis
from $x=0$ to $x=\pi/2$

$$\begin{aligned}\Rightarrow \text{Area} &= \int_a^b y \cdot dx \\ &= \int_{a=0}^{b=\pi/2} \sin x \cdot dx \\ &= \left[-\cos x \right]_0^{\pi/2} \\ &= -\cos\left(\frac{\pi}{2}\right) + \cos(0) \\ &= -\cos(90^\circ) + \cos(0^\circ) \\ &= 0 + 1 \\ \text{Area} &= 1 \\ &= 1 \text{ sq. units}\end{aligned}$$

$$\pi = 180^\circ$$

$$\frac{\pi}{2} = \frac{180^\circ}{2} = 90^\circ$$

Numerical Method

2M ~~*~~ 1) Bakhshali Method (k-scheme)

4M — { 2) Bisection Method

3) Regula-falsi Method (false position Method)

4M / 6M — 4) Newton Raphson Method

4M / 6M — 5) Jacobi's Method

4M / 6M — 6) Gauss Seidel Method

Bisection Method

Q. find real root of equation

$$x^3 - x - 4 = 0 \text{ using Bisection Method}$$

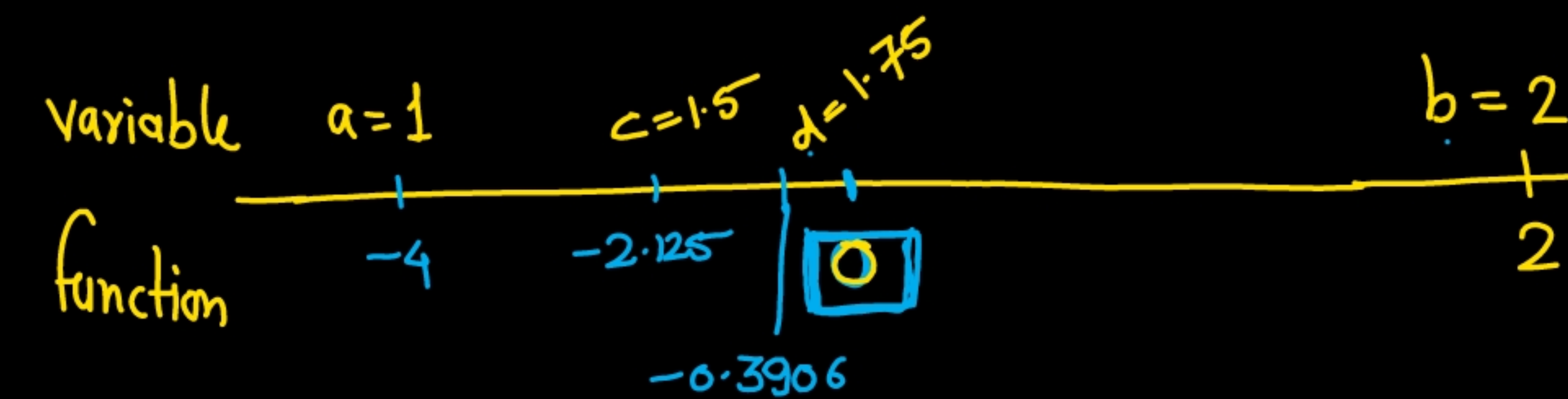
$\Rightarrow$

$$f(x) = x^3 - x - 4$$

$$\begin{aligned} \text{if } x=0 \\ f(0) &= 0^3 - 0 - 4 \\ &= -4 < 0 \end{aligned}$$

$$\begin{aligned} \text{if } x=1 \\ f(1) &= 1^3 - 1 - 4 \\ &= -4 < 0 \end{aligned}$$

$$\begin{aligned} \text{if } x=2 \\ f(2) &= 2^3 - 2 - 4 \\ &= 2 > 0 \end{aligned}$$



1st Iteration

$$c = \frac{a+b}{2} = \frac{1+2}{2}$$

$$c = 1.5$$

$$\begin{aligned} f(c) &= x^3 - x - 4 \\ &= 1.5^3 - 1.5 - 4 \\ &= -2.125 < 0 \end{aligned}$$

2nd Iteration

$$d = \frac{(c+b)}{2} = \frac{(1.5+2)}{2}$$

$$d = 1.75$$

$$\begin{aligned} f(d) &= x^3 - x - 4 \\ &= 1.75^3 - 1.75 - 4 \\ &= -0.3906 \end{aligned}$$

3rd Iteration

$$\begin{aligned} e &= \frac{(d+b)}{2} \\ &= \frac{(1.75+2)}{2} = 1.875 \end{aligned}$$

$$\begin{aligned} f(e) &= x^3 - x - 4 \\ &= 1.875^3 - 1.875 - 4 \\ &= 0.7168 \end{aligned}$$

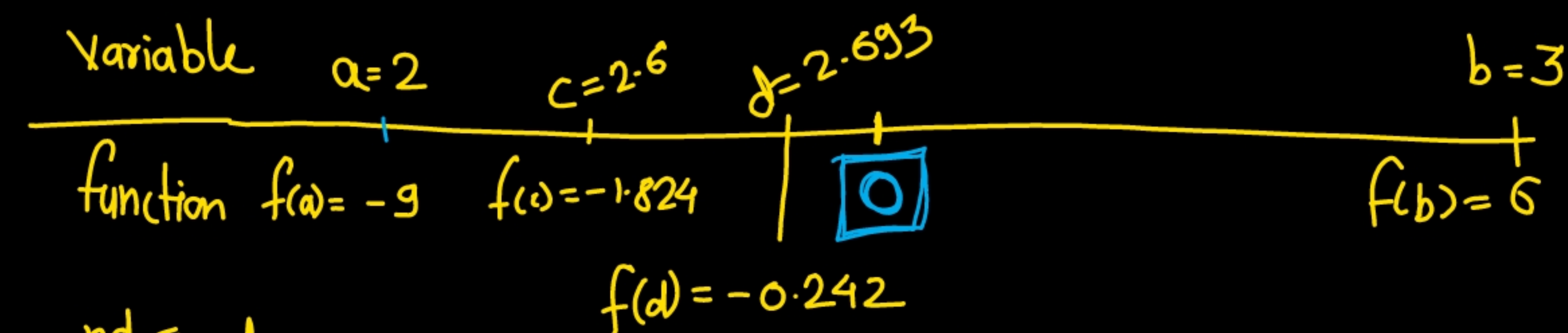
4M

✓ $\therefore$ Root of function $x^3 - x - 4 = 0$ is $e = 1.875 //$

Regula-falsi Method

Q. Find root of eqⁿ $x^3 - 4x - 9 = 0$
in interval $(2, 3)$ by using false position Method

$$\Rightarrow \begin{array}{l|l} a=2 & b=3 \\ f(a)=x^3-4x-9 & f(b)=x^3-4x-9 \\ =2^3-4 \times 2-9 & =3^3-4 \times 3-9 \\ =-9 < 0 & =6 > 0 \end{array}$$



1st Iteration

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$
$$= \frac{2 \times 6 - 3 \times (-9)}{6 - (-9)}$$

$$c = 2.6$$

$$f(c) = x^3 - 4x - 9$$
$$= 2.6^3 - 4 \times 2.6 - 9$$

$$f(c) = -1.824$$

2nd Iteration

$$d = \frac{c \cdot f(b) - b \cdot f(c)}{f(b) - f(c)}$$
$$= \frac{2.6 \times 6 - 3 \times (-1.824)}{6 - (-1.824)}$$

$$d = 2.693$$

$$f(d) = x^3 - 4x - 9$$
$$= 2.693^3 - 4 \times 2.693 - 9$$

$$f(d) = -0.242 //$$

3rd Iteration

$$e = \frac{d \cdot f(b) - b \cdot f(d)}{f(b) - f(d)}$$

$$e = \frac{2.693 \times 6 - 3 \times (-0.242)}{6 - (-0.242)}$$

$$e = 2.705$$

$$f(e) = x^3 - 4x - 9$$
$$= 2.705^3 - 4 \times 2.705 - 9$$
$$= -0.027$$

$\therefore$ Root of eqⁿ $x^3 - 4x - 9 = 0$ is $e = 2.705 //$

Newton Raphson Method

Q. find $\sqrt[3]{100} = ?$ using Newton-Raphson Method

$$\Rightarrow \begin{aligned} x &= \sqrt[3]{100} \\ x &= (100)^{1/3} \\ \text{taking cube on both sides} \\ x^3 &= [(100)^{1/3}]^3 \\ x^3 &= 100 \end{aligned}$$

$$f(x) = x^3 - 100 = 0 \quad \text{--- (1)}$$

differentiating w.r.t 'x'

$$\begin{aligned} f'(x) &= \frac{d}{dx}(x^3 - 100) \\ &= \frac{d}{dx}x^3 - \frac{d}{dx}100 \\ &= 3x^2 - 0 \end{aligned}$$

$$f'(x) = 3x^2 \quad \text{--- (2)}$$

$$f(x) = x^3 - 100 \quad \text{--- (1)}$$

$$* \quad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\sqrt[3]{100} = 4.6415$$

$$\approx 4.642$$

$$x_0 = 4$$

$$\sqrt[3]{100} = (100)^{1/3}$$

$$= (100)^{1 \div 3}$$

$$\approx 4.642 //$$

1st Iteration $n=0, x_0=4$

$$x_{0+1} = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 4 - \frac{(-36)}{48}$$

$$x_1 = 4.75$$

$$\begin{aligned} f(x) &= x^3 - 100 & f'(x) &= 3x^2 \\ f(x_0) &= x_0^3 - 100 & f'(x_0) &= 3 \times 4^2 \\ &= 4^3 - 100 & &= 48 \\ &= -36 & & \end{aligned}$$

2nd Iteration $n=1, x_1=4.75$

$$x_{1+1} = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 4.75 - \frac{7.172}{67.688}$$

$$x_2 = 4.644 //$$

$$\begin{aligned} f(x) &= x^3 - 100 & f'(x) &= 3x^2 \\ f(x_1) &= x_1^3 - 100 & f'(x_1) &= 3 \times x_1^2 \\ &= 4.75^3 - 100 & &= 3 \times 4.75^2 \\ &= 7.172 & &= 67.688 \end{aligned}$$

3rd Iteration $n=2, x_2=4.644$

$$x_{2+1} = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 4.644 - \frac{0.156}{64.7}$$

$$= 4.6418$$

$$x_3 \approx 4.642 //$$

$$\therefore \sqrt[3]{100} = 4.642 //$$

Jacobi Method

Q. $2x + 10y + z = 13$ ——— ① ✓

$10x + y + 2z = 12$ ——— ② ✓

$2x + 2y + 10z = 14$ ——— ③

⇒

Coeff. of 'x' is Max. in eqⁿ ②

$$10x + y + 2z = 12$$

$$10x = 12 - y - 2z$$

$$x = \frac{(12 - y - 2z)}{10}$$

Coeff. of 'y' is Max. in eqⁿ ①

$$2x + 10y + z = 13$$

$$10y = 13 - 2x - z$$

$$y = \frac{(13 - 2x - z)}{10}$$

eqⁿ ③ 'z'

$$2x + 2y + 10z = 14$$

$$10z = 14 - 2x - 2y$$

$$z = \frac{(14 - 2x - 2y)}{10}$$

1st Iteration

$$x_0 = 0, y_0 = 0, z_0 = 0$$

$$x_1 = \frac{(12 - y - 2z)}{10} = \frac{(12 - 0 - 2 \times 0)}{10}$$

$$x_1 = 1.2$$

$$y_1 = \frac{(13 - 2x - z)}{10} = \frac{(13 - 2 \times 0 - 0)}{10}$$

$$y_1 = 1.3$$

$$z_1 = \frac{(14 - 2x - 2y)}{10} = \frac{(14 - 2 \times 0 - 2 \times 0)}{10}$$

$$z_1 = 1.4$$

$$x_1 = 1.2, y_1 = 1.3, z_1 = 1.4$$

2nd Iteration

$$x_1 = 1.2, y_1 = 1.3, z_1 = 1.4$$

$$x_2 = \frac{(12 - y - 2z)}{10} = \frac{(12 - 1.3 - 2 \times 1.4)}{10}$$

$$x_2 = \frac{7.9}{10} = 0.79$$

$$y_2 = \frac{(13 - 2x - z)}{10} = \frac{(13 - 2 \times 1.2 - 1.4)}{10}$$

$$y_2 = 0.92$$

$$z_2 = \frac{(14 - 2x - 2y)}{10} = \frac{(14 - 2 \times 1.2 - 2 \times 1.3)}{10}$$

$$z_2 = 0.9$$

$$x_2 = 0.79, y_2 = 0.92, z_2 = 0.9$$

3rd Iteration

$$x_2 = 0.79, y_2 = 0.92, z_2 = 0.9$$

$$x_3 = \frac{(12 - y - 2z)}{10} = \frac{(12 - 0.92 - 2 \times 0.9)}{10}$$

$$= \frac{9.28}{10} = 0.928$$

$$y_3 = \frac{(13 - 2x - z)}{10} = \frac{(13 - 2 \times 0.79 - 0.9)}{10}$$

$$= \frac{10.52}{10} = 1.052$$

$$z_3 = \frac{(14 - 2x - 2y)}{10} = \frac{(14 - 2 \times 0.79 - 2 \times 0.92)}{10}$$

$$= \frac{10.58}{10} = 1.058 //$$

Solution $x_3 = 0.928$

$$y_3 = 1.052$$

$$z_3 = 1.058$$

Gauss-Seidel Method

Q $x + 4y + 2z = 15$ — ① ✓

$x + 2y + 5z = 20$ — ②

$5x + 2y + z = 12$ — ③ ✓

$$\Rightarrow \begin{array}{l|l|l} \text{Coeff. of 'x' is max. in eqn ③} & \text{Coeff. of 'y' is max. in eqn ①} & \text{eqn ②} \\ \hline 5x + 2y + z = 12 & x + 4y + 2z = 15 & x + 2y + 5z = 20 \\ 5x = 12 - 2y - z & 4y = 15 - x - 2z & 5z = 20 - x - 2y \\ x = \frac{(12 - 2y - z)}{5} & y = \frac{(15 - x - 2z)}{4} & z = \frac{(20 - x - 2y)}{5} \end{array}$$

1st Iteration

$x_0 = 0, y_0 = 0, z_0 = 0$

$x_1 = \frac{(12 - 2y - z)}{5} = \frac{(12 - 2 \times 0 - 0)}{5}$

$x_1 = 2.4$

$x_1 = 2.4, y_0 = 0, z_0 = 0$

$y_1 = \frac{(15 - x - 2z)}{4} = \frac{(15 - 2.4 - 2 \times 0)}{4}$

$y_1 = 3.15$

$x_1 = 2.4, y_1 = 3.15, z_0 = 0$

$z_1 = \frac{(20 - x - 2y)}{5} = \frac{(20 - 2.4 - 2 \times 3.15)}{5}$

$z_1 = 2.26$

$x_1 = 2.4, y_1 = 3.15, z_1 = 2.26$

2nd Iteration

$x_1 = 2.4, y_1 = 3.15, z_1 = 2.26$

$x_2 = \frac{(12 - 2y - z)}{5} = \frac{(12 - 2 \times 3.15 - 2.26)}{5}$

$x_2 = 0.688, y_1 = 3.15, z_1 = 2.26$

$y_2 = \frac{(15 - x - 2z)}{4} = \frac{(15 - 0.688 - 2 \times 2.26)}{4}$

$y_2 = 2.448$

$x_2 = 0.688, y_2 = 2.448, z_1 = 2.26$

$z_2 = \frac{(20 - x - 2y)}{5} = \frac{(20 - 0.688 - 2 \times 2.448)}{5}$

$z_2 = 2.8832$

$\therefore x_2 = 0.688, y_2 = 2.448, z_2 = 2.883 //$

I - Scheme

- 1) Function — 2 Mark / 4 Marks
- 2) Derivative —
- 3) Application of Derivative — } 20 Marks //

Today's session target = 24 Marks //

I - Scheme

Function

- 1) find value of function
- 2) find function is even or odd.

find value of $f(5)$
if $f(x) = x^2 - 2x + 5$

$$\Rightarrow \begin{aligned} f(x) &= x^2 - 2x + 5 \\ f(5) &= 5^2 - 2 \times 5 + 5 \\ &= 20 // \end{aligned}$$

Q. find $\underline{f(5) + f(-2)}$
if $f(x) = x^3 - 3x + 4$

$$\Rightarrow \begin{aligned} f(x) &= x^3 - 3x + 4 \\ f(5) &= 5^3 - 3 \times 5 + 4 \\ &= 114 \\ f(-2) &= (-2)^3 - 3(-2) + 4 \\ &= 2 \end{aligned}$$

$$\begin{aligned} f(5) + f(-2) &= 114 + 2 \\ &= 116 // \end{aligned}$$

Q. find $\underline{f(\frac{1}{2})}$ if $f(x) = 2^x - \log_2 x$

$$\begin{aligned} \Rightarrow f(x) &= 2^x - \log_2 x \\ f\left(\frac{1}{2}\right) &= 2^{\left(\frac{1}{2}\right)} - \log_2 \left(\frac{1}{2}\right) \\ &= 1.414 - \left[\log_2 1 - \log_2 2 \right] \quad \text{--- } \log\left(\frac{a}{b}\right) = \log a - \log b \\ &= 1.414 - [0 - 1] \\ &= 1.414 + 1 \\ f\left(\frac{1}{2}\right) &= 2.414 // \end{aligned}$$

Logarithm

$$1) \log(a \times b) = \log a + \log b$$

$$2) \log\left(\frac{a}{b}\right) = \log a - \log b$$

$$\# 3) \log a^x = x \cdot \log a$$

$$\# 4) \log_{\underline{a}} \underline{a} = 1$$

$$\# 5) \log_x 1 = 0$$

$$f(-x) = f(x)$$

→ even function

$$f(-x) = -f(x)$$

→ odd function

$$f(-x) \neq f(x)$$

$$\neq -f(x)$$

→ Neither even
Nor odd

$$\sin(x) \Rightarrow (-x) \Rightarrow \sin(-x) = -\sin x$$

$$\otimes \cos x \Rightarrow (-x) \Rightarrow \cos(-x) = \cos x$$

$$\tan x \Rightarrow (-x) \Rightarrow \tan(-x) = -\tan x$$

$$\cot x \Rightarrow (-x) \Rightarrow \cot(-x) = -\cot x$$

$$\otimes \sec x \Rightarrow (-x) \Rightarrow \sec(-x) = \sec x$$

$$\operatorname{cosec} x \Rightarrow (-x) \Rightarrow \operatorname{cosec}(-x) = -\operatorname{cosec} x$$

Q. state whether function is even or odd

$$f(x) = 3x^4 - 2x^2 + \cos x$$

⇒

$$f(x) = \underline{3x^4} - \underline{2x^2} + \underline{\cos x}$$

$$f(-x) = 3(-x)^4 - 2(-x)^2 + \cos(-x)$$

$$= \underline{3x^4} - \underline{2x^2} + \underline{\cos x} //$$

$$\textcircled{*} f(-x) = f(x)$$

↳ ∴ function is Even //

Q. state function is even or Odd

$$f(x) = 4x^4 + 3\cos x + x \cdot \sin x + 1$$

⇒

$$f(x) = \underline{4x^4} + \underline{3\cos x} + \underline{x \cdot \sin x} + \underline{1}$$

$$f(-x) = 4(-x)^4 + 3\cos(-x) + (-x) \cdot \sin(-x) + 1$$

$$= 4x^4 + 3\cos x + (-x) \cdot (-\sin x) + 1$$

$$= \underline{4x^4} + \underline{3\cos x} + \underline{x \cdot \sin x} + \underline{1}$$

$$f(-x) = f(x)$$

↳ ∴ function is even //

Q. find function is even or odd.

$$f(x) = x \cdot \cos x + \sin x$$

⇒

$$f(x) = \underline{+x \cdot \cos x} + \underline{\sin x}$$

$$f(-x) = (-x) \cdot \cos(-x) + \sin(-x)$$

$$= (-x) \cdot \cos x + (-\sin x)$$

$$f(-x) = \underline{-x \cdot \cos x} + \underline{-\sin x}$$

$$f(-x) = -f(x)$$

↳ ∴ function is odd //

Q. state whether function is even or odd.

$$f(x) = x^2 + x^3$$

⇒

$$f(x) = \underline{x^2} + \underline{x^3}$$

$$f(-x) = (-x)^2 + (-x)^3$$

$$= \underline{x^2} - \underline{x^3}$$

$$f(-x) \neq f(x)$$

$$\neq -f(x)$$

↳ ∴ function is Neither even
Nor odd //

DERIVATIVE

(Variable) ~~Constant~~

1) $\frac{d}{dx} \text{ constant} = 0$

2) $\frac{d}{dx} x^n = n \cdot x^{n-1}$

3) $\frac{d}{dx} kx^n = k \cdot \frac{d}{dx} x^n$

4) $\frac{d}{dx} a^x = a^x \cdot \log_e a$

5) $\frac{d}{dx} e^x = e^x$

6) $\frac{d}{dx} \log x = \frac{1}{x}$

7) $\frac{d}{dx} \sin x = \cos x$

8) $\frac{d}{dx} \cos x = -\sin x$

9) $\frac{d}{dx} \tan x = \sec^2 x$

10) $\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$

11) $\frac{d}{dx} \sec x = \sec x \cdot \tan x$

12) $\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cdot \cot x$

13) Product Rule

$$\frac{d}{dx} (u \cdot v) = u \cdot \frac{d}{dx} v + v \cdot \frac{d}{dx} u$$

14) Division Rule

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \cdot \frac{d}{dx} u - u \cdot \frac{d}{dx} v}{v^2}$$

15) $\frac{d}{dx} (u \pm v \mp w) = \frac{d}{dx} u \pm \frac{d}{dx} v \mp \frac{d}{dx} w$

16) Chain Rule

$$\frac{dy}{dx} = \frac{\frac{dy}{dz}}{\frac{dx}{dz}}$$

17) $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

18) $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$

19) $\frac{d}{dx} \sec^{-1} x = \frac{1}{|x| \sqrt{x^2-1}}$

20) $\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$

21) $\frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2}$

22) $\frac{d}{dx} \operatorname{cosec}^{-1} x = \frac{-1}{|x| \sqrt{x^2-1}}$

Derivative Sum

(2M) 1) $u \times v$
or $\frac{u}{v}$

(4M) 2) Chain Rule

(4M) 3) Implicite function
 $f(x, y) = 0$

(4M) 4) (Variable)^{variable}

(4M) 5) Parametric functions

Q. find $\frac{dy}{dx}$ if $y = x^3 \tan x$

$$y = u \times v$$

$$\Rightarrow y = x^3 \tan x$$

diff. w.r.t 'x'

$$\frac{d}{dx} y = u \cdot \frac{d}{dx} v + v \cdot \frac{d}{dx} u$$

$$= x^3 \frac{d}{dx} \tan x + \tan x \cdot \frac{d}{dx} x^3$$

$$= x^3 (\sec^2 x) + \tan x \cdot 3x^{2-1}$$

$$\frac{dy}{dx} = x^3 (\sec^2 x) + \tan x \cdot 3x^2 //$$

$$\left\{ \begin{array}{l} \frac{d}{dx} \tan x = \sec^2 x \\ \frac{d}{dx} x^n = n \cdot x^{n-1} \end{array} \right\}$$

Q. find $\frac{dy}{dx}$ if $y = \frac{\cot(2x)}{3^x} \cdot \frac{u}{v}$

$$\Rightarrow y = \frac{\cot(2x)}{3^x} \cdot \frac{u}{v}$$

diff. w.r.t 'x'

$$\frac{d}{dx} y = \frac{v \cdot \frac{d}{dx} u - u \cdot \frac{d}{dx} v}{v^2}$$

$$= \frac{3^x \frac{d}{dx} \cot(2x) - \cot(2x) \frac{d}{dx} 3^x}{(3^x)^2}$$

$$= \frac{3^x \frac{d}{dx} \cot x - \cot(2x) \cdot 3^x \cdot \log 3}{(3^x)^2} \quad \left\{ \frac{d}{dx} a^x = a^x \cdot \log a \right\}$$

$$= \frac{3^x \cdot (-\operatorname{cosec}^2 x) \cdot \frac{d}{dx} x - \cot(2x) \cdot 3^x \cdot \log 3}{(3^x)^2} \quad \left\{ \frac{d}{dx} \cot x = -\operatorname{cosec}^2 x \right\}$$

$$= \frac{3^x \cdot [-\operatorname{cosec}^2(2x)] \cdot \frac{d}{dx} (2x) - \cot(2x) \cdot 3^x \cdot \log 3}{(3^x)^2} \quad \left\{ \frac{d}{dx} x^n = n \cdot \frac{d}{dx} x^{n-1} \right\}$$

$$= \frac{3^x \cdot [-\operatorname{cosec}^2(2x)] \cdot 2 \cdot \left(\frac{d}{dx} x \right) - \cot(2x) \cdot 3^x \cdot \log 3}{(3^x)^2} \quad \left\{ \begin{array}{l} \frac{d}{dx} x^n = n \cdot x^{n-1} \\ \frac{d}{dx} x^1 = 1 \cdot x^{1-1} \\ = 1 \cdot x^0 \\ = 1 \cdot 1 = 1 \end{array} \right\}$$

$$\frac{dy}{dx} = \frac{3^x \cdot [-\operatorname{cosec}^2(2x)] \cdot 2 - \cot(2x) \cdot 3^x \cdot \log 3}{(3^x)^2}$$

Chain Rule

Q. $x = a \sin \theta$ & $y = a(1 + \cos \theta)$
find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$y = a(1 + \cos \theta)$$

diff. w.r.t θ

$$\frac{d}{d\theta} y = \frac{d}{d\theta} [a(1 + \cos \theta)]$$

$$= a \cdot \frac{d}{d\theta} [1 + \cos \theta]$$

$$= a \left[\frac{d}{d\theta} 1 + \frac{d}{d\theta} \cos \theta \right]$$

$$= a [0 + (-\sin \theta)]$$

$$\frac{dy}{d\theta} = -a \sin \theta$$

$$x = a \sin \theta$$

diff. w.r.t θ

$$\frac{d}{d\theta} x = \frac{d}{d\theta} a \sin \theta$$

$$= a \cdot \frac{d}{d\theta} \sin \theta$$

$$\frac{dx}{d\theta} = a \cdot \cos \theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a \sin \theta}{a \cdot \cos \theta}$$

$$= \frac{-\sin \theta}{\cos \theta} //$$

$$\therefore \frac{dy}{dx} = \frac{-\sin \theta}{\cos \theta}$$

$$\left. \frac{dy}{dx} \right|_{\text{at } \theta = \frac{\pi}{3}} = \frac{-\sin(60^\circ)}{\cos(60^\circ)}$$

$$= -\sqrt{3} //$$

$$= -1.732 //$$

$$\pi = 180^\circ$$

$$\frac{\pi}{3} = \frac{180}{3} = 60^\circ //$$

Q. find $\frac{dy}{dx}$ if $x^2 + y^2 - 8x + 4y + 12 = 0$
at point $(2, 0)$

$\Rightarrow$

$$x^2 + y^2 - 8x + 4y + 12 = 0$$

differentiating w.r.t x

$$\frac{d}{dx}(x^2 + y^2 - 8x + 4y + 12) = \frac{d}{dx} 0$$

$$\frac{d}{dx} x^2 + \frac{d}{dx} y^2 - \frac{d}{dx} 8x + \frac{d}{dx} 4y + \frac{d}{dx} 12 = 0$$

$$2x + \frac{d}{dx} x^2 - 8 \frac{d}{dx} x + 4 \cdot \frac{d}{dx} y + 0 = 0$$

$$2x + 2x \cdot \frac{d}{dx} x - 8(1) + 4 \cdot \frac{dy}{dx} = 0$$

$$2x + 2y \cdot \frac{dy}{dx} - 8 + 4 \cdot \frac{dy}{dx} = 0$$

$$2y \cdot \frac{dy}{dx} + 4 \cdot \frac{dy}{dx} = -2x + 8$$

$$\frac{dy}{dx} (2y + 4) = -2x + 8$$

$$\frac{dy}{dx} = \frac{-2x + 8}{2y + 4}$$

$$\left. \frac{dy}{dx} \right|_{\text{at } (2, 0)} = \frac{-2 \times 2 + 8}{2 \times 0 + 4}$$

$x \quad y$

$$= \frac{-4 + 8}{4}$$

$$\left. \frac{dy}{dx} \right|_{\text{at } (2, 0)} = 1 //$$

(Variable) ^{variable} $\rightarrow$ HINT - step ① take log on both sides //

Q. find $\frac{dy}{dx}$ if $y = (\tan x)^x$

$\Rightarrow$

$$\frac{d}{dx} x^n = \frac{d}{dx} (\text{Variable})^{\text{const.}} = n \cdot x^{n-1}$$

$$\frac{d}{dx} a^x = \frac{d}{dx} (\text{constant})^{\text{variable}} = a^x \cdot \log a$$

⊗ (Variable)^{variable} $\rightarrow$ take 'log' on both sides

$$y = (\tan x)^x$$

taking log on both sides

$$\log y = \log (\tan x)^x$$

$$\log y = x \cdot \log (\tan x)$$

differentiating w.r.t x

$$\frac{d}{dx} \log y = u \cdot \frac{d}{dx} v + v \cdot \frac{d}{dx} u$$

$$\frac{d}{dx} \log x_1 = x \cdot \frac{d}{dx} \log \tan x + \log (\tan x) \cdot \frac{d}{dx} x$$

$$\frac{1}{x_1} \cdot \frac{d}{dx} x_1 = x \cdot \frac{d}{dx} \log x + \log (\tan x) \cdot 1$$

$$\frac{1}{y} \cdot \frac{d}{dx} y = x \cdot \frac{1}{x} \cdot \frac{d}{dx} x + \log (\tan x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{\tan x} \cdot \frac{d}{dx} \tan x + \log (\tan x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{\tan x} (\sec^2 x) + \log (\tan x)$$

$$\frac{dy}{dx} = y \left[\frac{x}{\tan x} \cdot \sec^2 x + \log (\tan x) \right]$$

$$\frac{dy}{dx} = y \left[\frac{x}{\tan x} \cdot \sec^2 x + \log (\tan x) \right] //$$

$$\frac{dy}{dx} = (\tan x)^x \left[\frac{x}{\tan x} \cdot \sec^2 x + \log (\tan x) \right] //$$

4M

Q. find $\frac{dy}{dx}$ if $y = \tan^{-1} \left[\frac{2x}{1+35x^2} \right]$

$\Rightarrow$

$$y = \tan^{-1} \left[\frac{2x}{1+35x^2} \right]$$

$$y = \tan^{-1} \left[\frac{7x-5x}{1+(7x)(5x)} \right]$$

$$= \tan^{-1} \left[\frac{\tan(7x) - \tan(5x)}{1 + (\tan(7x) \cdot \tan(5x))} \right]$$

$$= \tan^{-1} \left[\tan(A-B) \right]$$

$$= \tan^{-1} \left[\tan(7x-5x) \right]$$

$$= (7x-5x)$$

$$y = 2x$$

diff. w.r.t. x

$$\frac{d}{dx} y = \frac{d}{dx} (2x)$$

$$\frac{dy}{dx} = 2 \left(\frac{d}{dx} x \right)$$

$$= 2 \times 1$$

$$\frac{dy}{dx} = 2 //$$

$$\frac{35x^2}{7x} - \frac{5x}{5x} = (2x) \text{ Nu.}$$

$$\tan A = \tan 7x$$

$$\tan B = \tan 5x$$

Application Derivative

- 1) find eqⁿ of Tangent & Normal
- 2) find Radius of Curvature
- 3) find Maxima & Minima //

1) finding eqⁿ of Tangent & Normal.

$\Rightarrow$

step ① find slope of tangent

$$\frac{dy}{dx} = m_1 = \tan \theta$$

step ② find eqⁿ of Tangent

a) Point (x_1, y_1)

b) slope = m_1

$$(y - y_1) = m_1 (x - x_1)$$

step ③ find slope of Normal

$$\begin{aligned} \text{slope of Tangent} \times \text{slope of Normal} &= -1 \\ m_1 \times m_2 &= -1 \end{aligned}$$

step ④ find eqⁿ of Normal

a) Point (x_1, y_1)

b) slope = m_2

$$(y - y_1) = m_2 (x - x_1) //$$

Q. Find eqⁿ of Tangent & Normal to curve $y = x(2-x)$ at $(2, 0)$

⇒

$$y = x(2-x)$$

$$y = 2x - x^2$$

step ① :- finding slope of Tangent $\left(\frac{dy}{dx}\right)$

$$y = 2x - x^2$$

diff. w.r.t 'x'

$$\frac{d}{dx} y = \frac{d}{dx} (2x - x^2)$$

$$= \frac{d}{dx} 2x - \frac{d}{dx} x^2$$

$$= 2 \frac{d}{dx} x - 2x$$

$$= 2(1) - 2x$$

$$\frac{dy}{dx} = 2 - 2x$$

$$\left. \frac{dy}{dx} \right|_{\text{at } (2,0)} = 2 - 2x$$

$$= 2 - 2 \times 2$$

$$m_1 = -2 //$$

↳ slope of tangent

step ② finding eqⁿ of Tangent

a) Point $(x_1, y_1) = (2, 0)$

b) slope = $m_1 = -2$

eqⁿ of tangent

$$(y - y_1) = m_1 (x - x_1)$$

$$(y - 0) = (-2)(x - 2)$$

$$y = (-2)x - 2(-2)$$

$$y = -2x + 4$$

$$2x + y - 4 = 0$$

↳ eqⁿ of tangent //

$$Ax + By + C = 0$$

step ③ finding slope of Normal

$$\text{slope of tangent} \times \text{slope of Normal} = -1$$

$$m_1 \times m_2 = -1$$

$$(-2) \times m_2 = -1$$

$$m_2 = \frac{-1}{(-2)}$$

$$m_2 = 0.5$$

↳ slope of Normal

step ④ finding eqⁿ of Normal

a) Point $(x_1, y_1) = (2, 0)$

b) slope = $m_2 = 0.5$

Eqⁿ of Normal

$$(y - y_1) = m_2 (x - x_1)$$

$$(y - 0) = 0.5(x - 2)$$

$$y = 0.5x - 2 \times 0.5$$

$$y = 0.5x - 1$$

$$-0.5x + y + 1 = 0$$

↳ eqⁿ of Normal //

$$Ax + By + C = 0$$

Application of Derivative

② Radius of Curvature

step ① finding $\left(\frac{dy}{dx}\right)$

step ② finding $\left(\frac{d^2y}{dx^2}\right)$

step ③ find Radius of Curvature

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

Q. find Radius of Curvature to curve $y^2 = 4ax$
at point $(a, 2a)$

⇒ step ① finding $\left(\frac{dy}{dx}\right)$

$$y^2 = 4ax$$

differentiating w.r.t x

$$\frac{d}{dx} y^2 = \frac{d}{dx} 4ax$$

$$\frac{d}{dx} x^2 = 4a \cdot \frac{d}{dx} x$$

$$2x \cdot \frac{d}{dx} x = 4a \cdot 1$$

$$2y \cdot \frac{d}{dx} y = 4a$$

$$\frac{dy}{dx} = \frac{4a}{2y}$$

$$\left. \frac{dy}{dx} \right|_{\text{at } (a, 2a)} = \frac{4a}{2(2a)}$$

$$= \frac{4a}{4a}$$

$$\left. \frac{dy}{dx} \right|_{\text{at } (a, 2a)} = 1 //$$

step ② finding $\frac{d^2y}{dx^2}$

$$\frac{dy}{dx} = \frac{4a}{2y}$$

$$\frac{dy}{dx} = \frac{2a}{y}$$

diff. w.r.t x

$$\frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} \frac{2a}{y}$$

$$\frac{d^2y}{dx^2} = 2a \cdot \frac{d}{dx} \frac{1}{y}$$

$$\frac{d^2y}{dx^2} = 2a \cdot \frac{d}{dx} y^{-1}$$

$$= 2a \cdot \frac{d}{dx} x^{-1}$$

$$= 2a \left[(-1) \cdot x^{-1-1} \cdot \frac{d}{dx} x \right]$$

$$= 2a \left[(-1) y^{-2} \cdot \frac{d}{dx} y \right]$$

$$\frac{d^2y}{dx^2} = (-2a) \cdot \frac{1}{y^2} \cdot \frac{dy}{dx}$$

$$\left. \frac{d^2y}{dx^2} \right|_{\text{at } (a, 2a)} = (-2a) \cdot \frac{1}{(2a)^2} \cdot \left(\frac{dy}{dx} \right)_{\text{at } (a, 2a)}$$

$$= (-2a) \cdot \frac{1}{4a^2} \cdot 1$$

$$= -2a \cdot \frac{1}{4a^2}$$

$$\left. \frac{d^2y}{dx^2} \right|_{\text{at } (a, 2a)} = -\frac{1}{2a} //$$

step ③

$$\text{Radius of Curvature } (\rho) = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$= \frac{\left[1 + (1)^2 \right]^{1.5}}{\left(-\frac{1}{2a} \right)}$$

$$= \frac{2.828}{\left(-\frac{1}{2a} \right)}$$

$$= \frac{2.828}{1} \times \frac{2a}{-1}$$

$$\rho = -5.657a \text{ units} //$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c}$$

Application of Derivative

③ Maxima & Minima

step ① finding $\frac{dy}{dx}$

step ② find value of x if $\frac{dy}{dx} = 0$

step ③ find value of $\frac{d^2y}{dx^2}$

step ④ finding Maxima & Minima

$\frac{d^2y}{dx^2} = \text{negative}$

$\rightarrow \therefore$ function is Maxima

$\frac{d^2y}{dx^2} = \text{positive}$

$\rightarrow$ function is Minima //

Q. find Maxima & Minima

$$\text{for } x^3 - 9x^2 + 24x$$

$\Rightarrow$

$$y = x^3 - 9x^2 + 24x$$

step ① $\Rightarrow$

finding $\frac{dy}{dx}$

$$y = x^3 - 9x^2 + 24x$$

diff. w.r.t x

$$\frac{d}{dx} y = \frac{d}{dx} x^3 - \frac{d}{dx} 9x^2 + \frac{d}{dx} 24x$$

$$= 3x^2 - 9 \frac{d}{dx} x^2 + 24 \cdot \frac{d}{dx} x$$

$$\frac{dy}{dx} = 3x^2 - 9(2x) + 24(1)$$

$$\frac{dy}{dx} = 3x^2 - 18x + 24$$

step ② find value of x if $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 0 = 3x^2 - 18x + 24$$

$$x_1 = 4 \quad \& \quad x_2 = 2$$

step ③

finding $\frac{d^2y}{dx^2}$

$$\frac{dy}{dx} = 3x^2 - 18x + 24$$

differentiating w.r.t ' x '

$$\frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} 3x^2 - \frac{d}{dx} 18x + \frac{d}{dx} 24$$

$$\frac{d^2y}{dx^2} = 3 \cdot \frac{d}{dx} x^2 - 18 \frac{d}{dx} x + 0$$

$$= 3(2x) - 18(1)$$

$$\frac{d^2y}{dx^2} = 6x - 18$$

step ④ :-

$$x_1 = 4$$

$$\left. \frac{d^2y}{dx^2} \right|_{\text{at } x=4} = 6x - 18$$

$$= 6 \times 4 - 18$$

$$= 6$$

$\therefore \frac{d^2y}{dx^2}$ is positive

$\therefore$ function is Minima

$$y_{\text{minima}} = x^3 - 9x^2 + 24x$$

$$= 4^3 - 9 \times 4^2 + 24 \times 4$$

$$y_{\text{mini.}} = 16 //$$

$$x_2 = 2$$

$$\left. \frac{d^2y}{dx^2} \right|_{\text{at } x=2} = 6x - 18$$

$$= 6 \times 2 - 18$$

$$= -6$$

$\therefore \frac{d^2y}{dx^2}$ is negative

function is Maxima

$$y_{\text{maxima}} = x^3 - 9x^2 + 24x$$

$$= 2^3 - 9 \times 2^2 + 24 \times 2$$

$$y_{\text{max.}} = 20 //$$